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Enhanced Estimation of Reference Evapotranspiration using Hybrid Deep Learning Models and Remote Sensing Variables

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Abstract

Effective water resources management and irrigation scheduling for agricultural sector highly depend on the precise estimation of reference evapotranspiration, ET_0. Both long short-term memory (LSTM) and gated recurrent unit (GRU) showed their equivalent capability in estimating ET_0 and achieved the highest R^2 and lowest prediction errors. Hybrid deep learning models, CNN-LSTM and CNN-GRU managed to improve the accuracy of the prediction. Incorporation of surface reflectance bands and auxiliary variables enhanced the performance of the models. This study provides valuable insights into deep learning algorithms and further confirms the potential of remote sensing variables as an alternative data source for ET_0 estimation.

Keywords: MODIS Aqua, Long Short-Term Memory, Gated Recurrent Unit

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1 Introduction

Evapotranspiration (ET) comprises two main processes, namely transpiration and evaporation that describe the loss of water from the surface of the crop plants and the land surface, respectively, into the atmosphere. The indirect ET estimate technique based on reference evapotranspiration (ET_0) and crop coefficient has been widely adopted for ET

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estimation. The present study aims to develop the ET_0 estimation models using deep learning algorithms, based on significant remote sensing variables and estimated daytime LST in Peninsular Malaysia.

2 Method

2.1 Study Area and Data

A total of six meteorological parameters, namely mean temperature (T_{mean}), maximum temperature (T_{max}), minimum temprature (T_{min}), wind speed at an altitude of 2 m (u_2), relative humidity (RH), and solar radiation (R_s), were collected for Pulau Langkawi and Kuantan stations. The collected meteorological data were used to calculate the ET_0 using Penman-Monteith (PM) equation (Equation 2.1), which was proposed and recommended by Food and Agriculture Organisation (FAO) (Allen et al., 1998). Remote sensing data such as daytime land surface temperature (LST), normalized difference vegetation index (NDVI), enhanced vegetation index (EVI), surface reflectance bands, solar zenith angle (SZA) offered by MODIS/Aqua and downward shortwave radiation (DSR) from MODIS Terra and Aqua, were used in this study.

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma\frac{900}{T+273}u_2(e_s - e_a)}{\Delta + y(1 + 0.34u_2)} \tag{2.1}$$

where Δ is the slope of saturation vapour pressure ($kPa^{\circ}C^{-1}$), y is the psychometric constant, $(e_s - e_a)$ is the actual vapour pressure deficit (kPa), G is the soil heat flux ($MLm^{-2}day^{-1}$), T is the daily mean temperature ($^{\circ}C$), u_2 is the wind speed at $2m$ height (m/s) and R_n is the net radiation ($MLm^{-2}day^{-1}$).

2.2 Models Development and Performance Evaluation

Two ET_0 estimation models have been developed using LSTM and GRU based on remote sensing variables such as DSR, surface reflectance bands and daytime LST estimated from ground and remote sensing datasets. SVR was chosen as the regression model for the daytime LST estimation in this study due to its additional distinct advantages in regression tasks (Abobakr Yahya et al., 2019) and has gained broad use in predictive modeling with multiple variables (Alnuwaiser et al., 2022). Both LSTM and GRU were chosen for the ET_0 estimation for the reason of their well performance in handling the sequence learning tasks Pyo et al. (2023), Shen et al. (2018). The dataset was split with 80% allocated for training, 10% for validation and 10% for testing. ET_0 calculated using FAO-PM method were used as target datasets and the estimation models were trained on various input combinations. Hybrid models, CNN-LSTM and CNN-GRU were developed to enhance the accuracy of the estimation.

3 Results

The results revealed that both LSTM and GRU were competent in connecting the input variables to the ET_0. Both models showed equivalent capability in estimating ET_0, achieving the highest R^2 values of 0.695 and 0.796, the lowest MAE values of 0.440 and 0.268, and the lowest RMSE values of 0.565 and 0.350, at the Pulau Langkawi and Kuantan stations, respectively. Improved estimation accuracy was achieved by the hybrid models

CNN-LSTM and CNN-GRU, which attained the highest R^2 values of 0.713 and 0.805, the lowest MAE values of 0.427 and 0.257, the lowest RMSE values of 0.548 and 0.343, at the Pulau Langkawi and Kuantan stations, respectively.

Table 1: Results of ET_0 estimation.

Station	**Pulau Langkawi**			**Kuantan**		
Scenario (Best)	3			2		
Model	**MAE**	**RMSE**	R^2	**MAE**	**RMSE**	R^2
LSTM	0.440	0.565	0.695	0.278	0.358	0.787
GRU	0.446	0.566	0.694	0.268	0.350	0.796
CNNLSTM	0.429	0.554	0.706	0.257	0.347	0.800
CNNGRU	0.427	0.548	0.713	0.265	0.343	0.805

4 Conclusions

In this study, LSTM and GRU have been established to predict the ET_0 at Pulau Langkawi and Kuantan stations. The experimental results showed that both models demonstrated equivalent capability in estimating ET_0 using remote sensing variables. Hybrid models managed to improve the accuracy by increasing the R^2 and lowering the prediction errors. ET_0 estimation using DSR and estimated LST may not be sufficient. Incorporation of other remote sensing variables significantly improved the performance of the estimation models, indicating their indispensable roles in ET_0 estimation.

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